HEAT SOURCE DENSITY AND TEMPERATURE DISTRIBUTION IN A LASER RUBY CRYSTAL

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The heat source density in a laser ruby is calculated by numerical integration of the pumping and absorption spectra. The heat source density can be represented approximately by the formula $q(r) = q_1 + q_2 I_0(\xi r/r_0)$, which can be used to determine the temperature distribution in the crystal for typical pumping and cooling conditions.

We consider a long ruby crystal of round section with a constant absorption coefficient $k(\lambda')$ throughout its volume and a smooth, nonmat surface on which isotropic pumping radiation falls. The heat source density (specific power of the sources) in the crystal can be calculated from the formula

$$q(r) = \int_{\lambda_1}^{\lambda_2} E(\lambda')F(\lambda', k, r)k(\lambda')\left[1-\eta(\lambda')\frac{\lambda'}{\lambda_{21}}\right]d\lambda'. \quad (1)$$

In formula (1) $E(\lambda')$ is the illuminance of the crystal surface, which depends on the selected pumping lamp; F is a factor which takes into account the distribution of pumping light in the crystal; η is the quantum yield of luminescence and $\lambda_{21} = 0.69\mu$ is the average wavelength of the luminescent radiation. Formula (1) ignores secondary absorption of the luminescent emission, the change in the value of $k(\lambda')$ due to depletion of the ground level, and the change in quantum yield due to heating.

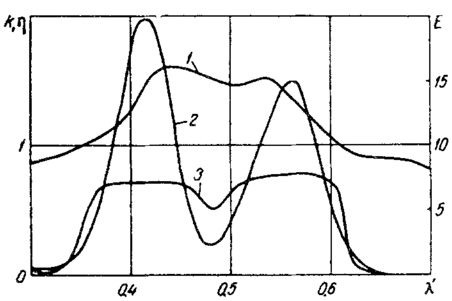

Fig. 1. 1) Spectrum $E(\lambda')$ (rel. units) of IFP-800 pumping lamp; 2) absorption spectrum $k(\lambda')$ (cm^{-1}) of ruby crystal; 3) quantum yield of luminescence $\eta(\lambda')$ (dimensionless units) (λ' in μ).

The value of q(r) is determined from formula (1) by numerical integration with a step $\Delta\lambda' = 0.01\mu$ and the following initial data (Fig. 1): the spectrum E (λ') of a IFP-800 xenon lamp recorded in pulsed operation with a pumping energy of 200 J and voltage 800 V: the absorption spectrum obtained for a ruby crystal with chromium concentration 0.05% on a SF-10 spectrophotometer after averaging over the two polarizations: factor F as a function of the parameters Kr_0 and r/r_0, taken from Anan'ev and Korolev's paper [1] on the distribution of monochromatic pumping radiation in laser crystals; and, finally the quantum yield $\eta(\lambda')$, chosen to correspond with Bukke and Morgenshtern's measurements [2], the curve given in [2] being continued into the 0.58- to 0.65 μ region, as shown in Fig. 1 (curve 3). The chosen diameter of the crystal in the calculations was $2r_0 = 0.65$ cm. We assumed that filters are used to isolate the region from $\lambda_1 = 0.3$ to $\lambda_2 = 0.7\mu$ in the pumping spectrum and that the rest of the spectrum is not implicated in the heating of the crystal.

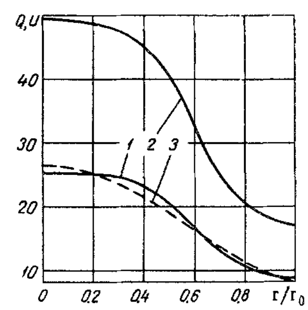

Fig. 2. 1) Density of energy specific heat energy Q(r) (J/cm^3) released during one pumping pulse in a ruby of diameter 6.5 mm; 2) absorbed energy of pumping radiation U (r) (J/cm^3); 3) approximate representation of specific energy of sources by means of function $Q(r) = Q_1 + Q_2 I_0(\xi r/r_0)$ with $Q_1 = 12.5$ J/cm^3 and $Q_2 = 14$ J/cm^3, $\xi = 3.2$.

The results of calculation of q(r) are given in Fig. 2 (curve 1). The curve is normalized so that the total amount of heat Q released in the crystal during one pumping pulse is 40 J, and the density of the heat released during a pulse Q(r) is plotted on the y axis. The determined distribution of source density is far from homogeneous. We also calculated the distribution of absorbed pumping power

$$p(r) = \int_{\lambda_1}^{\lambda_2} E(\lambda')F(\lambda', k, r)k(\lambda')d\lambda' \quad (2)$$

and the energy absorbed during a pumping pulse U(r) (curve 2, Fig. 2). We can then evaluate the energy yield of luminescence η' from the absorbed energy:

$$\eta' = \left\{\int_0^{r_0}[p(r)-q(r)]dr\right\}\Big/\left\{\int_0^{r_0} p(r)dr\right\}. \quad (3)$$

In this case it is $\eta' = 0.49$.

The determined heat source density can be represented by the approximate formula

$$q(r) = q_1 + q_2 I_0(\xi r/r_0). \tag{4}$$

Figure 2 (curve 3) shows a function of the form (4) – the density of the heat released during a pulse with $Q_1 = 12.5$, $Q_2 = 14$ J/cm^3, and $\xi = 3.2$; this represents the value of Q(r) to an accuracy of ±10%. The selection of an approximate formula in the form (4) in conjunction with a Bessel function facilitates the evaluation of the integrals in the calculation of the temperature distribution.

Below we calculate the temperature distribution in a homogeneous and isotropic laser crystal in the case of periodic pulsed pumping with heat source density of the form (4). We assume that the ends of the crystal are thermally insulated and on the cylindrical surface boundary conditions of the first or third kind with a constant heat transfer coefficient are satisfied. We assume that at the instant of application of the first pumping pulse the crystal temperature T(r, 0) is equal to the temperature of the cooling medium T_0, i.e., $\Theta(r, 0) = T(r, 0) - T_0 = 0$. It is known that this boundary value problem can be solved for any q (r, t) by Hankel's method of finite integral transforms [3]. For this purpose the periodic function of time q (r, t) must be put in the form of a Fourier series. It is much more convenient, however, to express the source density by a simple piecewise-continuous function assigned to an arbitrary pumping cycle $0 < \tau < t_c$. Then the problem is divided into a series of stages corresponding to the first, second, etc., cycles of operation. The temperature at the end of the (m − 1)th cycle is the initial condition for the next, m-th cycle. The successive solution of these problems leads to the following formula for the temperature distribution during the m-th cycle of operation:

$$\Theta_m(r,\tau) = \frac{a}{\lambda}\sum_{n=1}^{\infty} L_n^{-1} I_0(p_n r)\exp(-ap_n^2\tau) \times \left\{\int_0^{\tau} \bar{q}(p_n,\tau')\exp(ap_n^2\tau')\,d\tau' + \frac{1-\exp[-(m-1)ap_n^2 t_c]}{\exp(ap_n^2 t_c)-1} \times \int_0^{t_c} \bar{q}(p_n,\tau')\exp(ap_n^2\tau')\,d\tau'\right\}. \tag{5}$$

In formula (5) $\bar{q}(p_n,\tau) = \int_0^{r_0} rq(r,\tau)\, I_0(p_n r)\,dr$ is the Hankel image of the source density, p_n are the roots of the characteristic equation and $L_n = \int_0^{r_0} r I_0^2(p_n r)\,dr$. The temperature distribution in the following problems is calculated by means of formula (5).

Periodic pumping by rectangular and instantaneous pulses. During one cycle of operation the source density has value (4) in the interval $0 < \tau < t_0$; $t_c = t_p + t_0$. For heat transfer on the surface in accordance with Newton's law the temperature distribution in the course of the m-th cycle of operation is equal on heating to

$$\Theta_{mH}(r,\tau) = \frac{1}{\lambda}\sum_{n=1}^{\infty} A_n I_0(\beta_n r)\left\{\frac{q_1}{\beta_n^2} + \frac{q_2}{\beta_n^2 - \xi^2/r_0^2}\left[I_0(\xi) - \frac{\xi}{\mathrm{Bi}} I_1(\xi)\right]\right\}[1 - \varphi_{mH}(\beta_n)\exp(-a\beta_n^2\tau)] \tag{6}$$

and on cooling to

$$\Theta_{m0}(r,\tau) = \frac{1}{\lambda}\sum_{n=1}^{\infty} A_n I_0(\beta_n r)\left\{\frac{q_1}{\beta_n^2} + \frac{q_2}{\beta_n^2 - \xi^2 r_0^2}\left[I_0(\xi) - \frac{\xi}{\mathrm{Bi}} I_1(\xi)\right]\right\}\varphi_{m0}(\beta_n)\exp(-a\beta_n^2\tau). \tag{7}$$

In formulas (6) and (7) β_n are the roots of the characteristic equation $\beta r_0 I_1(\beta r_0) - \mathrm{Bi} I_0(\beta r_0) = 0$; A_n are coefficients tabulated in [4, 5],

$$A_n = 2\,\mathrm{Bi}/(\mathrm{Bi}^2 + \beta_n^2 r_0^2)\, I_0(\beta_n r_0).$$

The values of $\varphi_{mH}(\beta_n)$ and $\varphi_{m0}(\beta_n)$ are

$$\varphi_{mH}(\beta_n) = 1 - [\exp(-a\beta_n^2 t_0) - \exp(-a\beta_n^2 t_c)] \times \frac{1-\exp[-(m-1)a\beta_n^2 t_c]}{1-\exp(-a\beta_n^2 t_c)} \tag{8}$$

$$\varphi_{m0}(\beta_n) = \frac{1-\exp(-ma\beta_n^2 t_c)}{1-\exp(-a\beta_n^2 t_c)}[1 - \exp(-a\beta_n^2 t_H)] \tag{9}$$

and for the established periodic temperature conditions are converted to the limiting expressions $\varphi_{\infty H}(\beta_n) = \lim_{m\to\infty}\varphi_{mH}(\beta_n)$ and $\varphi_{\infty 0}(\beta_n) = \lim_{m\to\infty}\varphi_{m0}(\beta_n)$. If the heat transfer coefficient is very high and the boundary conditions can be written in the form $\Theta(r_0, t) = 0$, then heating and cooling during the m-th cycle of operation conform to the law

$$\Theta_{mH}(r,\tau) = \frac{2}{\lambda r_0}\sum_{n=1}^{\infty}\frac{I_0(\alpha_n r)}{\alpha_n I_1(\alpha_n r_0)} \times \left[\frac{q_1}{\alpha_n^2} + \frac{q_2}{\alpha_n^2 - \xi^2/r_0^2} I_0(\xi)\right][1 - \varphi_{mH}(\alpha_n)\exp(-a\alpha_n^2\tau)],$$
$$0 < \tau < t_H; \tag{10}$$

$$\Theta_{m0}(r,\tau) = \frac{2}{\lambda r_0}\sum_{n=1}^{\infty}\frac{I_0(\alpha_n r)}{\alpha_n I_1(\alpha_n r_0)}\left[\frac{q_1}{\alpha_n^2} + \frac{q_2}{\alpha_n^2 - \xi^2/r_0^2} I_0(\xi)\right] \times \varphi_{m0}(\alpha_n)\exp(-a\alpha_n^2\tau), \quad 0 < \tau < t_0, \tag{11}$$

where α_n are the roots of the characteristic equation $I_0(\alpha r_0) = 0$. Putting m = 1 and $t_0 \to \infty$ in formulas (6)–(11), we obtain expressions for a single rectangular pumping pulse. Formulas (6) and (10) with m = 1 and $\tau = t$ $(0 < t < \infty)$ also describe the establishment of steady-state heat conditions in the case of continuous and constant pumping of form (4).

In the case where the duration of the pumping pulses can be neglected we convert in formulas (7) and (11) to the limit $[q_1 + q_2 I_0(\xi r/r_0)]t_p \to Q_1 + Q_2 I_0(\xi r/r_0)$ where $t_p \to 0$. For a finite heat transfer coefficient instantaneous pumping pulses produce a temperature distribution

$$\Theta_m(r,\tau)=\frac{a}{\lambda}\sum_{n=1}^{\infty}A_nI_0(\beta_n r)\left\{Q_1+\frac{Q_2\beta_n^2}{\beta_n^2-\xi^2/r_0^2}\left[I_0(\xi)-\frac{\xi}{\mathrm{Bi}}I_0(\xi)\right]\frac{1-\exp(-ma\beta_n^2t_u)}{1-\exp(-a\beta_n^2t_u)}\exp(-a\beta_n^2\tau)\right\},\quad(12)$$

and with the zero boundary condition $\Theta(r_0, t)=0$ we have

$$\Theta_m(r,\tau)=\frac{2a}{\lambda r_0}\sum_{n=1}^{\infty}\frac{I_0(\alpha_n r)}{\alpha_n I_1(\alpha_n r_0)}\left[Q_1+\frac{Q_2\alpha_n^2}{\alpha_n^2-\xi^2/r_0^2}I_0(\xi)\right]\times\frac{1-\exp(-ma\alpha_n^2t_u)}{1-\exp(-a\alpha_n^2t_u)}\exp(-a\alpha_n^2\tau).\quad(13)$$

When $\tau=t_c$ formulas (12) and (13) give the temperature distribution in the crystal before the next instantaneous pumping pulse.

Bell-shaped pulses. In the case where no special measures are taken to shape the pumping pulses their shape may be far from rectangular and can be described fairly accurately by means of one, two or more time exponents. We consider successive identical pumping pulses with heat source density assigned to an arbitrary cycle of operation $0 < \tau < t_c$ by the function

$$q(r,\tau)=\sum_{s=1}^{S}[q_{1s}+q_{2s}I_0(\xi_s r/r_0)]\exp(-k_s\tau).\quad(14)$$

A function of the form (14) can be used to take into consideration the change in the pumping spectrum during the pulse. Substituting the value of (14) in formula (5) we obtain an expression for the temperature distribution during the m-th cycle of operation with a finite heat transfer coefficient

$$\Theta_m(r,\tau)=\frac{a}{\lambda}\sum_{s=1}^{S}\sum_{n=1}^{\infty}\frac{A_nI_0(\beta_n r)}{k_s-a\beta_n^2}\left\{q_{1s}+\frac{q_{2s}\beta_n^2}{\beta_n^2-\xi_s^2/r_0^2}\left[I_0(\xi_s)-\frac{\xi_s}{\mathrm{Bi}}I_1(\xi_s)\right]\right\}[\psi_{sm}(\beta_n)\exp(-a\beta_n^2\tau)-\exp(-k_s\tau)]\quad(15)$$

and with zero boundary condition

$$\Theta_m(r,\tau)=\frac{2a}{\lambda r_0}\sum_{s=1}^{S}\sum_{n=1}^{\infty}\frac{I_0(\alpha_n r)}{(k_s-a\alpha_n^2)I_1(\alpha_n r_0)}\times\left[\frac{q_{1s}}{\alpha_n}+\frac{q_{2s}\alpha_n}{\alpha_n^2-\xi_s^2/r_0^2}I_0(\xi_s)\right]\times[\psi_{sm}(\alpha_n)\exp(-a\alpha_n^2\tau)-\exp(-k_s\tau)].\quad(16)$$

The values of ψ_{sm} are

$$\psi_{sm}(p_n)=1+\frac{1-\exp[-(m-1)ap_n^2t_c]}{1-\exp(-ap_n^2t_c)}\times[\exp(-ap_n^2t_c)-\exp(-k_st_c)].\quad(17)$$

To investigate the steady-state periodic temperature conditions we merely convert to the limit where $m\to\infty$ in formulas (15)–(17). Putting $m = 1$ and $\tau = t$ $(0 < t < \infty)$ we obtain formulas for a case of a single pumping pulse of the form (14).

Finally, if the discharge circuit of the pulsed lamp has high inductance, the time dependence of the discharge current can be given [6] by the function $q(\tau) = q_0k\tau\exp(-k\tau)$. Let the source density $q(r_1, \tau)$ in the interval $0 < \tau < t_c$ have the value

$$q(r,\tau)=\sum_{s=1}^{S}[q_{1s}+q_{2s}I_0(\xi r/r_0)]k_s\tau\exp(-k_s\tau).\quad(18)$$

Then the temperature distribution during the m-th cycle is calculated from the formula

$$\Theta_m(r,\tau)=\frac{a}{\lambda}\sum_{s=1}^{S}\sum_{n=1}^{\infty}\frac{A_nI_0(\beta_n r)}{k_s-a\beta_n^2}\left\{q_{1s}+\frac{q_{2s}\beta_n^2}{\beta_n^2-\xi_s^2/r_0^2}\left[I_0(\xi_s)-\frac{\xi_s}{\mathrm{Bi}}I_1(\xi_s)\right]\right\}\left[\eta_{sm}(\beta_n)\exp(-a\beta_n^2\tau)-\left(k_s\tau+\frac{k_s}{k_s-a\beta_n^2}\right)\times\exp(-k_s\tau)\right],\quad(19)$$

where

$$\eta_{sm}(\beta_n)=\frac{k_s}{k_s-a\beta_n^2}+\frac{1-\exp[-(m-1)a\beta_n^2t_c]}{\exp(a\beta_n^2t_c)-1}\left[\frac{k_s}{k_s-a\beta_n^2}\left(1-\exp[(a\beta_n^2-k_s)t_c]\right)-k_st_c\exp[(a\beta_n^2-k_s)t_c]\right].\quad(20)$$

With the zero boundary condition the corresponding formula is obtained after replacement of the time factors in (16) by the values

$$\eta_{sm}(\alpha_n)\exp(-a\alpha_n^2\tau)-\left(k_s\tau+\frac{k_s}{k_s-a\alpha_n^2}\right)\exp(-k_s\tau).\quad(21)$$

It is obvious that in the case where the source density can be regarded as homogeneous, formulas (6), (7), (10)–(13), (15), (16), and (19) become simpler, since they lose the terms with q_2, which are zero.

NOTATION

r_0 is the radius of crystal; r is the variable radius (coordinate); t is the time; τ is the time during considered process or cycle of operation; $q(r, \tau)$ is the density (specific power) of heat sources; Q (r) is the specific energy (during pumping pulse) of heat sources; λ' is the wavelength of pumping radiation; $k(\lambda')$ is the absorption coefficient; F is the Anan'ev-Korolev factor, which takes into account the distribution of the monochromatic pumping radiation in the absorbing crystal; $\eta(\lambda')$ is the quantum yield of luminescence; p (r) is the absorbed power of pumping radiation; U (r) is the absorbed pumping energy; ξ is the empirical parameter in heat source distribution function; T (r, t) is the crystal temperature; T_0 is the temperature of cooling medium; $T(r, t) - T_0 = \Theta(r, t)$ is the excess of crystal temperature over ambient temperature; t_p is the duration of rectangular pumping pulses; t_0 is the time of cooling between rectangular pumping pulses; t_c is the duration of cycle of operation ($t_c = t_p + t_0$); λ is the thermal conductivity; a is the thermal diffusivity; α is the heat transfer coefficient; $Bi = \alpha r_0/\lambda$ is the Biot number; p_n, β_n, α_n are the roots of characteristic equations; A_n is the tabulated coefficients in problem of cooling of cylinder; I_0, I_1 are the Bessel functions of zero and first order; k_s is the index of time decay of heat sources; φ, ψ, η are the calculated coefficients in formulas for determining temperature distributions.

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